Backmixing in Multistage Agitated Contactors—a Correlation

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A generalized correlation is presented for the backmix ratio in multistage agitated contactors. The backmix ratio is shown to be a function of the ratio of impeller tip speed to superficial flow velocity, and of the column configuration and impeller design. For single-phase flow and for the continuous phase in two-phase flow, the following expression is recommended:

$$\alpha = 0.0098 \left[\epsilon \frac{ND}{u} \left(\frac{D^2 a}{THA} \right)^{1/2} \right]^{1.24}$$

Multistage agitated columns are becoming increasingly popular for liquid-liquid extraction, staged chemical reaction, and other, similar applications. The essential features of the device are illustrated by Figure 1. This design was first proposed by Oldshue and Rushton (*13*). Many variations of the basic concept are possible, depending on the particular application. For example, the settling chambers in Figure 1 are not required if a single fluid is being processed, or two fluids in concurrent flow.

Most applications need to take into account that the stages in a multistage agitated contactor (MAC) are not entirely separated, and that a certain amount of backmixing between stages is inevitable. Backmixing, as a rule, will lower the overall efficiency. The theoretical aspects of the problem have been discussed by a number of authors, notably Klinkenberg (*6*), Miyauchi and Vermeulen (*10*), and Sleicher (*17, 18*). Measurements of backmixing in MAC's have been reported by Yagi and Miyauchi (*20*), Nagata et al. (*12*), Gutoff (*4*), Bibaud and Treybal (*2*), and Miyauchi et al. (*11*).

These data were taken in units of various sizes and configurations, over a fairly wide range of flow rates and agitation intensities, and with two-phase as well as single-phase flow. In some cases special devices were installed for controlling the rate of backmixing, but most of the work was done on units basically conforming to the design of Figure 1. This author has collected some additional data which further extend the range of size and operating conditions. The purpose of the present paper is to show that all of the data presently available can be represented by a single generalized correlation.

MATHEMATICAL MODELS OF BACKMIXING

There are two basic models for describing backmixing, or longitudinal dispersion, in multistage units. The first treats the unit as a cascade of discrete, perfectly mixed stages, and assumes that in addition to the net forward flow of phase i, F_i, there is a backmix flow f_i between adjacent stages. For single-phase flow in the absence of chemical reaction, the material balance equation for any given species around the j^{th} stage is then

$$V_j \frac{dc_j}{dt} = (F + f)\, c_{j-1} - (F + 2f)\, c_j + f c_{j+1} \qquad (1)$$

Modifications needed to include more general cases, such as two-phase flow, mass transfer between phases, and chemical reaction, are obvious.

It is impossible, or impractical, to solve analytically the system of Equations (1) for nonsteady state situations if there are more than three stages. Still, these equations are the starting point in the determination of backmix flow rates from the results of tracer tests. If V_j, F, and f are constant, the variance of the residence-time distribution of a cascade containing n stages is found, using Laplace transforms and the method of moments, as

$$\sigma^2 = \frac{1 - x^2 - \dfrac{2x}{n}(1 - x^n)}{n(1 - x)^2} \qquad (2)$$

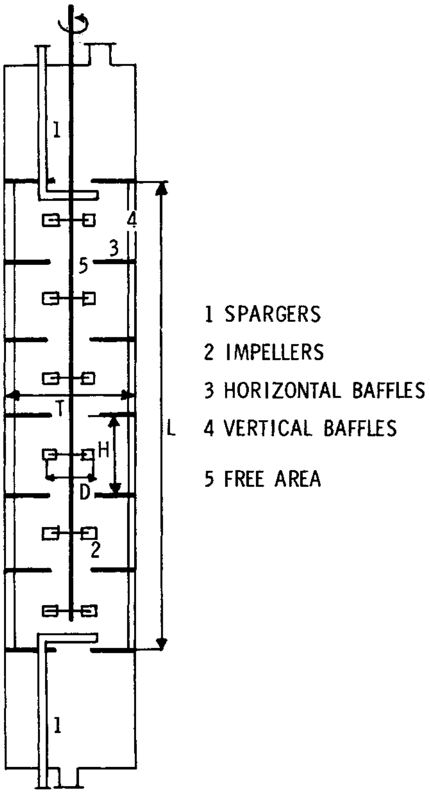

Fig. 1. Multistage agitated contactor.

TABLE 1. INVESTIGATIONS OF BACKMIXING IN SINGLE-PHASE FLOW

Author	T, cm.	H, cm.	a/A	N, r.p.m.	u, cm./min.	Liquid	Method	Reported	Symbol Fig. 2
Yagi et al. (*20*)	15	6 to 60	0.19			Water	Pulse	D_e	
Gutoff (*4*)	10	5	0.094 to 0.25	0 to 400	0.92 to 1.85	Water	Steady state	D_e, f	⊙
Bibaud et al. (*2*)	15	8.25	0.29	150 to 400	7.4 to 32	Water	Pulse	D_e	◆
Miyauchi et al. (*11*)	15 to 30	3 to 15	0.19 to 1.0	275 to 955		Water	No-flow mixing	f	
Miyauchi et al. (*11*)	15	7.5	0.18	85 to 1,350	0.25 to 4.4	Water	Pulse	f, D_e	●
Present work	15	7.5	0.045 to 0.135	0 to 1,200	10 to 61	Water	Pulse	α	+
Present work	15	7.5	0.135	500 to 1,200	20 to 40	Corn Syrup	Pulse	α	
Present work	240	85	0.16	21 to 54	8.4 to 20	Water/solution	Step	α	◬
Present work	120	74	0.034	155	10 to 37	Water/solution	Step	α	▽
Present work	15	9.3	0.0024 to 0.093	200 to 1,000	1.3 to 5.4	Water	Pulse	α	×

$$x = \frac{f}{F + f}$$

This solution was apparently first developed by Van der Laan and reported by Overcashier et al. (*14*). Once f is known, design equations of the type of Equation (1) can be solved numerically.

The other model that has been applied to multistage reactors or extractors replaces the backmix ratio by a dispersion coefficient, and the system of finite-difference equations by a single partial differential equation:

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial z} = D_e \frac{\partial^2 c}{\partial z^2} \tag{3}$$

In other words, the multistage unit is treated as a plug-flow device with superimposed axial dispersion. The variance of the residence-time distribution by this model is given by Van der Laan (*19*):

$$\sigma^2 = \frac{2D_e}{uL} \left\{ 1 - \frac{D_e}{uL} \left[1 - \exp \left(-\frac{uL}{D_e} \right) \right] \right\} \tag{4}$$

Both of these mathematical descriptions have been applied to MAC's. Each one has been used by some of the authors who have measured backmixing. The cascade model is more general because it is applicable to as few as two stages, for which the dispersion model is not an adequate description. A dye injection test on a transparent unit will convince most observers that the assumption of perfect mixing within each stage and of finite concentration jumps between stages is not at all unreasonable.

The correspondence between the two models and the relationship between backmix ratio and dispersion coefficient can be established in various ways. Gutoff (*4*) derived the following expression:

$$\frac{uH}{D_e} = \frac{n-1}{n} \ln \left(1 + \frac{1}{\alpha} \right) \tag{5}$$

This equation is applicable only for large n and α. At $\alpha = 0$, that is, in the case of an ideal cascade with no backmixing, Equation (5) predicts $D_e = 0$, which means perfect plug flow rather than staged flow. If Equation (3) is written in finite-difference form, using lowest order central differences, then by comparing terms with Equation (1)

$$\alpha = \frac{D_e}{uH} - \frac{1}{2} \tag{6}$$

Miyauchi et al. (*11*) use the formula

$$\alpha = \frac{D_e}{uL} - \frac{n}{2n-1} \tag{7}$$

which reduces to Equation (6) for large n.

Equation (3) clearly does not correctly describe a cascade of only a few stages, although it is found in practice that Equations (1) and (3) give virtually identical results in most situations for ten stages or more. Baldwin and Durbin (*1*), Roemer and Durbin (*15*), Miyauchi and Vermeulen (*10*), and Hartland and Mecklenburgh (*5, 7* to *9*) have examined this problem in considerable detail.

In the context of the present analysis, the problem is how to convert the results of tracer tests expressed as axial dispersion coefficients into interstage mixing rates. Bibaud and Treybal (*2*) and Miyauchi et al. (*11*) evaluated their results in terms of axial dispersion. They did so by fitting the pulse response curve, obtained by integrating Equation (3), to the data, using the match at the peak as sole criterion. In both cases 12-stage columns were used. This meant that the continuous model was a good approximation of the cascade, and a curve fitted at the peak matched the rest of the data quite well.

The conversion from axial dispersion to backmix ratio was accomplished for the two sets of data by setting the right-hand side of Equations (2) and (4) equal, that is, by calculating x to give a value for σ^2 equal to that obtained by Equation (4). This convenient method is justified by the obvious relationship between variance and peak height. Equation (7) gives practically identical values for α.

For multiphase operation the expression analogous to Equation (6) is

$$\alpha_i = \frac{f_i}{F_i} = \frac{D_{e,i}\, \phi_i}{u_i\, H} - \frac{1}{2} \tag{8}$$

where f_i is the backmix rate of the i^{th} phase. One may postulate that there should be an overall rate of exchange between stages f which should depend only on hydrodynamic conditions and be the same as in single-phase flow. The backmix rate of the i^{th} phase should be given approximately by

$$f_i = \phi_i\, f \tag{9}$$

Such a relationship may hold at least for liquid-liquid dispersions with small density differences. One may therefore attempt to establish a correspondence between single-phase and two-phase results at equal agitator speeds and flow rates according to

$$\alpha = \frac{\alpha_i}{\phi_i} \tag{10}$$

Deviations would most likely be caused by additional mixing induced by the movement of immiscible fluids relative to each other, especially in countercurrent flow.

Visual observation of flow patterns in countercurrent flow of immiscible liquids in MAC's shows that for the dispersed phase, one of the basic assumptions of the mathematical treatment, perfect mixing in each stage, is far from satisfied. The residence-time distribution is strongly influenced by such factors as preferential wetting of the wall and the occurrence of stagnant pockets above or below the horizontal baffles. For this reason, backmixing of the dispersed phase will not be considered here. Qualitatively, one may state the following: At low agitation intensities, where stagnant pockets may exist, the dispersed phase has a wider residence-time distribution than the continuous phase. At higher speeds, where the stagnant pockets disappear, the reverse is true.

These statements apply only to the case where the dispersed phase does not wet the walls. The agitator speed where the stagnant pockets are eliminated presumably depends on density difference, interfacial tension, etc.

SOURCES OF DATA

The various investigations and the range of principal variables have been summarized in Tables 1 and 2. Most of the work was done on pilot-plant size units of 15 cm. diam., but the largest column tested had a diameter of 240 cm. The free area—the opening in the horizontal baffles which divide the column into compartments—varied from 0.24 to 29%. Miyauchi et al. (*11*) performed some tests on a special no-flow unit with no horizontal baffles at all. Agitator speed varied from 0 to 1,350 r.p.m. The tests without agitation were omitted for the purposes of the present work, however. The width of the residence-time distribution in MAC's increases at very low agitator speeds, but this is caused by severe channeling rather than backmixing. Agitation power inputs ranged up to 50 hp./1,000 gal.

The superficial velocity in the single-phase tests ranged between 0.25 and 61 cm./sec. Water was used in all cases except for a few tests with corn syrup (viscosity 65 cp), designed to show the effect of viscosity. The large-scale measurements involved displacement of water by an aqueous solution. This method had to be chosen because of local plant limitations. It is not very precise because the tail section of the displacement curve is lost in the scatter of analytical measurements. The calculated backmix ratio tends to be too low. The error can be estimated by the procedure of Curl and McMillan (*3*). This method was applied to the plant data, resulting in an increase of about 30% in the α values.

Gutoff, as well as Miyauchi et al. in their special no-flow work, determined backmixing directly by following the spreading of a tracer within the column. All other investigators calculated it from residence-time distributions obtained by either pulse or step input techniques.

The free area between stages is equal to the central opening in the horizontal baffles, less the cross section of the agitator shaft. In some of Gutoff's tests and in the 120-cm. diam. column, the baffle opening was made larger than usual, and a disk was attached to the rotating shaft in the baffle plane, leaving an annular space for passage of the liquid.

Table 2 contains all the essential information on two-phase experiments. All of the data were taken in 15-cm. diam. columns, and the continuous phase in all cases was water. Only backmixing in the continuous phase has been considered for the purpose of the present work, although a few data on backmixing in the dispersed phase are available. Experiments on liquid-liquid and gas-liquid systems were included, in countercurrent as well as concurrent flow. Fractional holdup of the continuous phase was recorded in all cases, usually calculated from the flow rates and mean-residence times of the two phases.

The last column in Tables 1 and 2 contains the symbols which represent each set of data in Figures 2 and 3, respectively.

DATA CORRELATION

All of the correlations published to date are based in some way on the ratio of impeller tip speed to superficial velocity of the liquid. As the impeller speed increases, the eddy diffusivity or the rate of exchange across the free area between compartments increases, but it is the ratio of backflow to superficial velocity which appears in both Bodenstein number uH/D_e and backmix ratio α. Thus it seems natural to start from a relationship of the type

$$\alpha = \alpha\left(\frac{ND}{u}\right)$$

Backmixing must also be a function of the free area between stages, of the impeller size relative to height and diameter of each stage, and of the impeller design.

The first correlation tried here was

$$\alpha = k\left(\epsilon\frac{ND^3}{uTH}\right)^p\left(\frac{a}{A}\right)^q \tag{11}$$

ND^3 is a measure of the impeller pumping rate. TH is the vertical cross section of a compartment, and ND^3/TH then represents a mean velocity of circulation. ϵ is a correction

Table 2. Investigations of Backmixing in Two-Phase Flow

Author	H, cm.	a/A	N, r.p.m.	u_c, cm./min.	u_d, cm./min.	Disp. fluid	Flow arr.	Holdup cont. phase	Reported	Symbol Figure 3
Bibaud et al. (*2*)	8.75	0.29	150 to 250	7.4 to 15.5	8.2 to 22.0	Toluene	Countercurrent	0.73 to 0.97	D_e	◈
Bibaud et al. (*2*)	8.25	0.29	200 to 300	11.5 to 19.8	13.1 to 22.2	Kerosene	Countercurrent	0.77 to 0.93	D_e	
Miyauchi et al. (*11*)	7.5	0.18	240 to 900	0.85 to 2.15	1.13 to 3.4	MIBK	Countercurrent	0.39 to 0.98	f, De	●
Present author	7.5	0.045 to 0.135	500 to 1200	10.7 to 16.0	5.1 to 10.2	Kerosene	Cocurrent	0.57 to 0.83	α	×
Present author	7.5	0.135	200 to 300	10.7	10.2 to 20.5	Trichloroethylene	Countercurrent	0.85 to 0.88	α	◬
Present author	7.5	0.135	250 to 1200	21.3	16 to 80	Nitrogen	Cocurrent	0.86 to 0.96	α	+
Present author	7.5	0.135	500 to 1200	21.3	16 to 80	Nitrogen	Counter current	0.82 to 0.94	α	⊙
Present author	15	0.024 to 0.107	300 to 450	4.4 to 52	540 to 5600	Air	Cocurrent	0.51 to 0.90	α	⊡
Present author	9.3	0.0024 to 0.0093	200 to 1000	0.85 to 3.4	0.85 to 5.1	Air	Cocurrent	0.71 to 0.99	α	▽

Note: In all cases, the column diameter was 15 cm., the continuous phase was water, and the pulse method was applied.

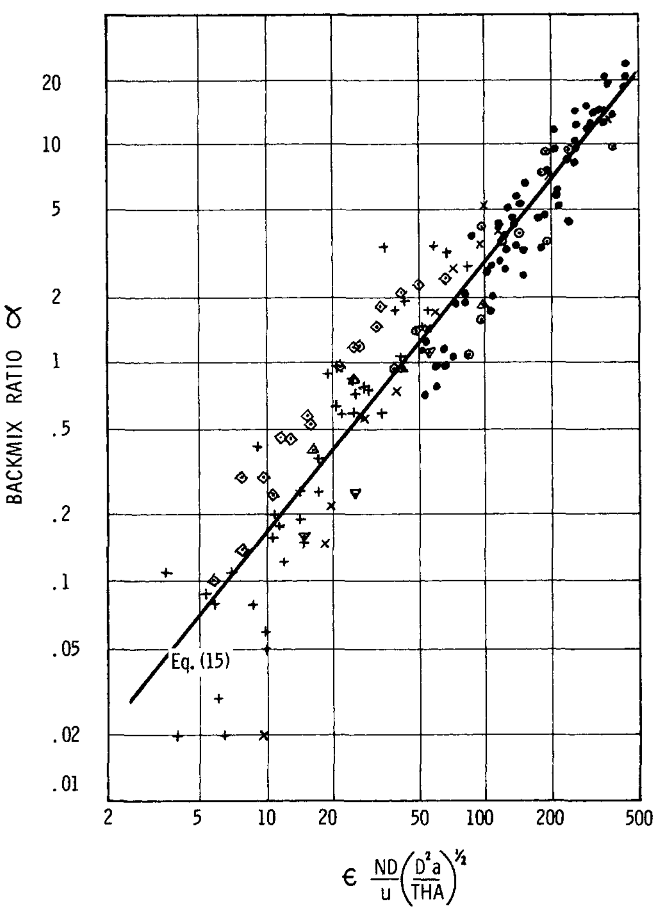

Fig. 2. Backmix ratio in single-phase flow.

factor which has to account for differences between impeller types. It has arbitrarily been given a value of 1.0 for six-blade turbines of the Rushton-Oldshue type. For other configurations, ϵ is taken as the ratio of the power number of the impeller used to that of the six-blade impeller. For turbines with more or less than six blades, the power number is approximately proportional to the square root of the number of blades.

Miyauchi et al. (*11*) propose that backmixing should be proportional to $N_p^{1/3}$ and compare results from a MAC and from a rotating disc contactor on this basis. In the course of the present work, a somewhat better correlation was obtained with $\epsilon \propto N_p$ than with $\epsilon \propto N_p^{1/3}$. However, the range of this variable was too small for a conclusive test.

The coefficient k and the exponents p and q in Equation (11) were determined by a least-squares fit, using all those sets of data in Table 1 showing a symbol in the last column. The total number of data points was 156. All points were given equal weight, and normal distribution of the logarithms with constant standard deviation was assumed. The second assumption was seen to be justified in the course of the regression analysis. Little is known about the reliability of the various sets of data and thus about the correctness of equal weighting.

With the results of the least-squares fit, Equation (11) becomes

$$\alpha = 0.0433 \left(\epsilon \frac{ND^3}{uTH} \right)^{1.178} \left(\frac{a}{A} \right)^{0.781} \quad (12)$$

Next, a simplified correlation was tried where the condition $p = 2q$ was imposed. The result was

$$\alpha = 0.0265 \left[\epsilon \frac{ND^3}{uTH} \left(\frac{a}{A} \right)^{1/2} \right]^{1.20} \quad (13)$$

The goodness of fit was somewhat better for Equation (12) than for (13). The residual error (square root of the error sum of squares divided by degrees of freedom) was 0.2312 for Equation (12), 0.2460 for Equation (13).

The data included in the correlation do not afford a meaningful test of the effect of the ratios D/T and D/H because these ratios varied only within narrow ranges. The only experiments with markedly different proportions were performed by Miyauchi et al. (*11*) in a no-flow unit. Miyauchi et al. obtained a good correlation by the equation

$$\frac{f}{a} = 0.027\, ND \left(\frac{T}{H} \right)^{1/2} \quad (14)$$

The general correlation was therefore modified to introduce the square root dependence on compartment height, and a new least-squares fit was performed, with the following result:

$$\alpha = 0.0098 \left[\epsilon \frac{ND}{u} \left(\frac{D^2 a}{THA} \right)^{1/2} \right]^{1.240} \quad (15)$$

The goodness of fit is better than for Equations (12) and (13) (residual error 0.2229).

A plot of Equation (15) and all data points is shown in Figure 2. Seventy-five percent of the points lie within a factor of 2 of the straight line. Deviations seem to be largest at very low backmix ratios. Accuracy is inherently lower in this range, as may be seen from a graph of Equation (2). A small change in σ^2 corresponds to a relatively large change in x. Since theoretical considerations at best give only qualitative guidance as to what kind of relationship should be expected, Equation (15) may be accepted as a satisfactory correlation of the available data.

Equation (14), which represents the no-flow results by Miyauchi et al., can be converted to the dimensionless

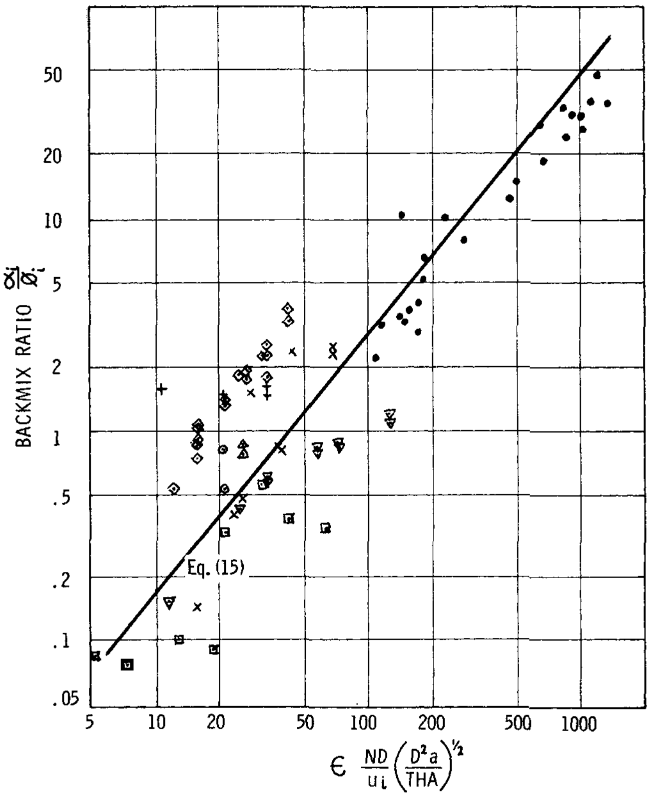

Fig. 3. Backmix ratio in continuous phase of two-phase flow.

Table 3. Backmixing in Single-Phase Flow of Corn Syrup Measured in 15-cm. diam. 12-stage column

Impeller Reynolds No.	$\frac{ND}{u}\left(\frac{D^2 a}{THA}\right)^{1/2}$	ϵ	α_{calc}	α_{obs}
430	21	0.61	0.24	0.38
1,060	51	0.69	0.81	1.02
444	10.8	0.61	0.10	0

form of Equation (15). First, by insertion of the various dimensions with their numerical values, it becomes

$$\frac{f}{a} = 0.05\ \epsilon\ ND \left(\frac{D^2}{TH}\ \frac{A}{a}\right)^{1/2} \tag{16}$$

Dividing by uA/a transforms this expression into

$$\frac{f}{uA} = \alpha = 0.05\ \epsilon\ \frac{ND}{u} \left(\frac{D^2}{TH}\ \frac{a}{A}\right)^{1/2} \tag{17}$$

Within the range of the no-flow data, this expression differs from Equation (15) by up to a factor of 2.

The results of Yagi and Miyauchi were not included here because of insufficient information. Miyauchi et al. show fair agreement between Yagi's data and their own.

The runs using corn syrup were compared with the rest of the single-phase runs under the assumption that the effect of viscosity could be accounted for by the pumping coefficient ϵ. A plot of the power factor versus impeller Reynolds number by Rushton et al. (*16*) was used to determine ϵ as a function of viscosity. The expected backmix ratio was then calculated from Equation (15) and compared with the observed value. The results in Table 3 show fair agreement.

Backmix ratios in the continuous phase of two-phase runs are plotted in Figure 3 as α/ϕ versus the same dimensionless number as in Figure 2. The velocity u is the superficial velocity of the continuous phase only. Equation (15) is also represented by the straight line in Figure 3. While the data qualitatively follow this line, deviations are distinctly greater than for single-phase runs. An attempt was made to relate the deviations to the flow rate and direction of the dispersed phase, but without success. No consistent effect of relative flow velocities was detected.

CONCLUSIONS

Backmixing in multistage agitated columns can be predicted over wide ranges of size, agitator speed, and flow rates by the generalized correlation, Equation (15). This correlation is valid for single-phase flow and, somewhat less accurately, for the dispersed phase of two-phase systems, either concurrent or countercurrent. Predicted values are accurate within about a factor of 2, which is adequate for most situations.

There is, of course, an almost infinite variety of special designs which would give tighter control of backmixing. The multistage agitated contactor is attractive because it is basically a very simple and versatile device. Its usefulness should be further enhanced by the proposed correlation which allows predicting backmix ratios and eliminates the need for model tests.

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NOTATION

A = column cross section, sq.cm.
a = free area in baffles between compartments, sq.cm.
c = concentration, g./cu.cm.
T = column diameter, cm.
D = impeller diameter, cm.
D_e = effective diffusivity, sq.cm./sec.
F = net flow rate, cu.cm./sec.
f = backflow rate, cu.cm./sec.
H = height of a compartment, cm.
k = numerical constant
L = length of column, cm.
N = agitator speed, sec.$^{-1}$
N_p = agitator power number
n = number of stages
p, q = undetermined exponents
t = time, sec.
u = superficial velocity, cm./sec.
V = volume of a compartment, cu.cm.
x = modified backmix ratio, $f/(F + f)$
z = length coordinate, cm.

Greek Letters

α = backmix ratio, f/F
ϵ = impeller correction factor
σ^2 = normalized variance of residence-time distribution
ϕ = fractional holdup

Subscripts

i = i^{th} phase
j = j^{th} stage

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